the single curve 1, while that for OT-4-1 is shown as curve 2 and that for VT-20 as curve 3), which indicates that the curves in these coordinates are invariant under change in the target thickness and projectile diameter throughout the speed range, not merely when a certain speed is attained.

Analogous plotting in the new coordinates was also used for the phenomenological $\Delta v(v_+)$ curves; the resulting $\Delta v/v_*(v_+/v_*)$ curves are shown in Fig. 4. The data form pronounced bundles for each material, which tend to asymptotes (1 for VT-20, 2 for OT-4-1, and 3 for D16T).

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EFFECTS OF STRAIN HISTORY ON THE DAMAGE ACCUMULATION RATE IN NONMONOTONE ELASTOPLASTIC LOADING

A. A. Movchan

UDC 539.4

1. The determination of the failure instant under conditions of nonmonotone elastoplastic loading may involve determining the number of cycles to failure in few-cycle fatigue [1, 2] and determining the plasticity reserve in complicated technological operations in pressure working of metals [3] as contrasting particular cases. Here we describe the accumulation of distributed damage, not the localized damage occurring after the formation of macroscopic cracks. Therefore, by the term failure we mean the generation of a crack of a certain fixed but small length. Although the superficial pictures of fatigue failure (few-cycle failure) and quasistatic failure are different [2], there are close similarities in the dislocation substructures, which define the damaged state of the material at the stage where delocalized damage accumulates [4], which indicates that a unified description may be possible. We assume that the ranges in strain rate and temperature are such that the choice of time scale is unimportant.

One way of providing a phenomenological description of damage accumulation is to introduce objects of scalar or tensor nature that describe the damage state. These objects are either specified as functionals of the loading path [5] or else their variations are defined by kinetic equations [6-8]. In [7, 8], the kinetic equation for the damage parameter Ω was written as

$$d\Omega/dL = \lambda P, \tag{1.1}$$

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where L is the length of the plastic-strain arc, P is the residual microstress intensity [9], and λ is a constant.

In [7, 10, 11], it was taken as "... a rational first approximation" [10] that P has a finite relation to the intensity of the plastic strain $p_i = (2/3)(p_{ij}p_{ij})^{1/2}$, where p_{ij} is the plastic-strain tensor. Then (1.1) becomes

$$d\Omega/dL = f(p_i). \tag{1.2}$$

From (1.2) one can describe few-cycle fatigue with strictly proportional strain in the half-cycles having a symmetrical cycle (zero mean strain) and for nonmonotone deformation with progressive increase in the plastic-strain intensity [7]. In [12, 13] it was shown that according to the criterion of (1.2) the damage in nonproportional cyclic strain is always greater than in cyclic strain of the same amplitude having proportional changes in the plastic-strain tensor components in the half-cycles, which has been confirmed by experiment [14]. In [15, 16], equations for few-cycles fatigue were derived that gave a qualitatively correct description of the experimental data for the superposition of strains orthogonal in space [17] on the harmonic plastic-strain paths superimposed with a phase shift or with different frequencies. It should be noted that in all the above cases where (1.2) describes the experimental data correctly, the means for the plastic-strain tensor components over a cycle were either constant and equal to zero or increased from cycle to cycle.

However, the applicability of (1.2) is restricted, since it does not incorporate the effects of the strain history on the damage accumulation rate. In [18] it was shown that the experimental data on few-cycle fatigue conflict with the assumption that the damage accumulated in a certain part of the strain path is independent of the history of the strain for any given damage summation law. This means that (1.2) will lead to a conflict for any function f. In fact, according to (1.2) the damage summation law is linear (i.e., is fixed); on integrating (1.2) over any part of the strain path, we get that the damage increment is dependent only on that part and is independent of the previous strain. In accordance with this, it was shown in [19] that (1.2) substantially overstates the effects of cycle asymmetry on the working life in few-cycle fatigue when the mean value of the strain parameter over a cycle is constant and different from zero, as it incorrectly describes the effects not only quantitatively but also qualitatively.

2. Here we incorporate the effects of strain history on the damage accumulation rate. Here (1.2) can be put as

$$\frac{d\Omega}{dL} = f\left[\sqrt{\frac{2}{3}}\,\rho(P, O)\right], \tag{2.1}$$

where $\rho(P, O)$ is the distance in strain space between the point O corresponding to the undeformed state and the point P representing the instantaneous strained one. Therefore, the distance in (2.1) is reckoned from the origin, and thus the asymmetry with respect to O has too large an effect on the working life. To avoid this, (2.1) may be rewritten as

$$\frac{d\Omega}{dL} = f\left[\sqrt{\frac{2}{3}}\,\rho(P, P_0)\right], \tag{2.2}$$

where P_0 is a certain distance reference point. During rigid cyclic strain, P moves twice over a certain rectilinear segment in strain space in each cycle, and then P_0 should approximate to the center of this segment. Then the effects of the asymmetry on the working life will decrease as the working life increases, as is observed in experiments [20]. This condition is satisfied by the center of mass P_1 on the strain path if the latter is considered as a constant-density curve in strain space. However, if P_0 and P_1 coincide, the effects of the asymmetry on the working life will be the same for different materials and will even be too small. Therefore, in what follows we assume that P_0 only approaches P_1 during the strain. The coordinates e^0_{ij} of P_0 and e^1_{ij} of P_1 are considered as related by $e^0_{ij} = \varphi(L)e^1_{ij}$, where $\varphi(L) \to 1$ for $L \to \infty$. In the subsequent calculations, $\varphi(L)$ is taken in the form $\varphi(L) = L/(L + L_*)$, where L_* is a certain constant. Then

$$e^0_{ij} = \frac{1}{L + L_*}\int_0^L p_{ij}(\xi)\,d\xi. \tag{2.3}$$

According to (2.3), $e^o_{ij} = e^1_{ij}$ for $L_* = 0$; the rate at which P_0 approaches P_1 decreases as L_* increases, and $e^o_{ij} \equiv 0$ for $L_* = \infty$.

In [19] it was shown that the phenomenon of allocyclic fatigue is described within the framework of (1.2) by the Coffin—Manson power law where f is a power law itself. Therefore, in that follows we take

$$f(x) = \lambda x^n. \tag{2.4}$$

As a result, the scalar parameter Ω and the tensor parameter ρ_{ij} describing the damage state are expressed by the following functionals:

$$\Omega = \int_0^L \lambda \left(\sqrt{\frac{2}{3}\rho_{ij}\rho_{ij}} \right)^n d\xi, \quad \rho_{ij} = p_{ij} - \frac{1}{L + L_*} \int_0^L p_{ij}(\xi)\, d\xi \tag{2.5}$$

or in differential form by

$$d\Omega = \lambda \left(\sqrt{\frac{2}{3}\rho_{ij}\rho_{ij}} \right)^n dL, \quad d\rho_{ij} = dp_{ij} - \frac{\rho_{ij}}{L + L_*} dL. \tag{2.6}$$

The λ, n, L_* of (2.5) and (2.6) are functions of the form of the state of stress and strain. System (2.6) is equivalent to (1.2) with (2.4) for $L_* = \infty$.

3. If there is proportional nonmonotone change in the plastic-strain tensor components

$$p_{ij} = p p^0_{ij}, \tag{3.1}$$

where p^o_{ij} is a constant deviator and p in general is a nonmonotonically varying parameter, then (2.5) can be reduced to

$$\Omega = \int_0^L \lambda \left\{ \frac{\left| pL_* + \int_0^l \operatorname{sign}\left(\frac{dp}{dL}\right) \xi d\xi \right|}{l + L_*} \right\}^n dl. \tag{3.2}$$

We apply (3.2) to the case of monotone strain and write the failure condition as

$$\Omega = 1, \tag{3.3}$$

to get a relationship between the model parameters λ, n, and L_*, which contains the plasticity D of the material (the intensity of the plastic strain at the instant of failure in monotone strain):

$$\lambda = \frac{1}{D^{n+1} \int_0^1 \left[\frac{2\xi\gamma + \xi^2}{2(\xi + \gamma)} \right]^n d\xi}, \tag{3.4}$$

where $\gamma = L_*/D$.

During cyclic strain, the distance reference point P_0 will move around a certain immobile point: the center of mass for the strain path for a cycle P_2 such that $\rho(P_0, P_2) \leq C/(Ai + B)$, where $\rho(P_0, P_2)$ is the maximum distance between P_0 and P_2 during a cycle, while A, B, and C are constants for a given cyclic strain path, with i the cyclic number. If the number of cycles is fairly large, the distance in (2.2) will be reckoned from a point P_0 close to the immobile point P_2, and the mode of damage accumulation comes close to stationary (the changes in the damage accumulation from cycle to cycle are small and decrease as the number of cycles increases). According to (2.2), the few-cycle fatigue equation for the stationary state of damage accumulation takes the form

$$N \int_0^{L_1} \lambda \left[\frac{2}{3} (p_{ij} - e^2_{ij})(p_{ij} - e^2_{ij}) \right]^{\frac{n}{2}} dL = 1, \tag{3.5}$$

where $e^2_{ij} = \int_0^{L_1} p_{ij}(\xi)\, d\xi$ are the coordinates of point P_2; the integration is taken along the strain arc during one cycle. Calculations show that N_1/N_2 tends to one in cyclic strain if the diameter of the path is reduced in a cycle, where N_1 and N_2 are the numbers of cycles to failure found correspondingly from (2.5), (3.3) and (3.5).

On comparing (1.2) with a power function f and (3.5) we conclude that (1.2) will give the same results as (3.5) under conditions of stationary damage accumulation, which means that it will give results similar to those obtained from (2.5), provided that $e^2_{ij} = 0$, i.e., when the components p_{ij} averaged over a cycle are zero. If this is not so, (1.2) will give results close to those from solving (2.5) for a sufficiently large number of cycles if by p_i we understand the intensity of the tensor $p_{ij} - e^2_{ij}$. Geometrically, this is equivalent to transferring the origin in strain space to the center of mass on the strain path for a cycle. With these reservations, the above results [7, 10-13, 15, 16] obtained from (1.2) are approximately correct within the framework of (2.5) if the number of loading cycles is sufficiently large.

In rigid cyclic strain, p in (3.1) varies cyclically between given limits of p_1 and p_2 ($p_2 > p_1$), and under conditions of stationary damage accumulation we get from (3.5) an equation for few-cycle fatigue in the form

$$\delta = 2\left(\frac{n+1}{4\lambda}\right)^{1/(n+1)} \frac{1}{N^{1/(n+1)}}, \qquad (3.6)$$

where $\delta = p_2 - p_1$ is the scale of the plastic strain and N is the number of cycles to failure. Equation (3.6) has the same structure as the Coffin-Manson power-law equation for few-cycle fatigue, which has been repeatedly tested by experiment [1, 2]. If the latter is taken in the form

$$\delta = p_*/N^\alpha, \qquad (3.7)$$

we get the following relationship between the model functions and the cyclic viscosity p_* and the parameter α in the few-cycle fatigue curve, which are readily determined by experiment:

$$1/(n+1) = \alpha; \qquad (3.8)$$

$$2/(4\lambda\alpha)^\alpha = p_*. \qquad (3.9)$$

We substitute (3.4) into (3.9) to get

$$\left\{\frac{n+1}{2}\int_0^1\left(\frac{2\xi\gamma+\xi^2}{\xi+\gamma}\right)^n d\xi\right\}^{\frac{1}{n+1}} = \frac{p_*}{D}. \qquad (3.10)$$

Equations (3.8)-(3.10) solve the problem of deriving the model parameters in terms of the experimentally determined quantities D, α, and p_*. Here L_* is found graphically from (3.10), where the left side is the monotone function $\gamma = L_*/D$.

Equation (3.10) shows that p_*/D is an increasing function of L_* and n. For n = const, p_*/D increases from $2^{-\alpha}$ to $2^{1-2\alpha}$ as L_* increases from zero to infinity. With $L_* = \infty$ and $\alpha = 0.5$ we get from (3.10) that

$$p_* = D. \qquad (3.11)$$

Equation (3.11) was derived from (1.2) in [7] and was recommended on the basis of the experimental data of [1]. With $L_* = 0$ and $\alpha = 0.5$, from (3.10) we get

$$p_* = D/\sqrt{2}. \qquad (3.12)$$

Equation (3.12) was derived from other considerations in [21], where results are given derived from experimental data for various materials, which confirm (3.12).

In [2], it was recommended that one should use $p_* = D/2$ for few-cycle fatigue in structural components, which corresponds to n = 0 and which gives the minimal value of p_*/D for $L_* \geqslant 0$, $n \geqslant 0$.

Therefore, (2.5) has an advantage over (1.2) that the ratio of the cyclic viscosity p_* to the ordinary viscosity D is not constant but may vary from material to material, as is observed by experiment.

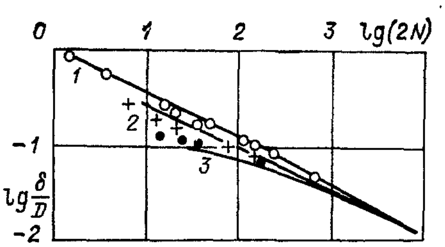

Fig. 1

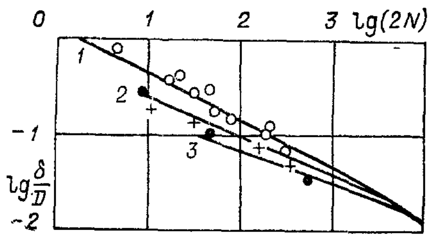

Fig. 2

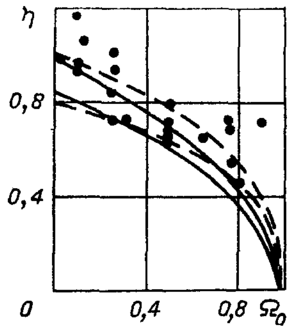

Fig. 3

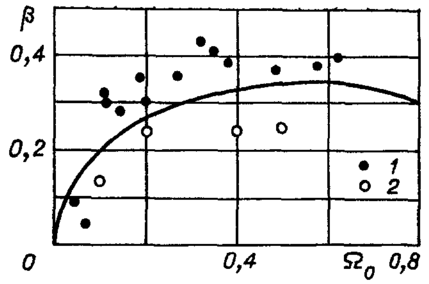

Fig. 4

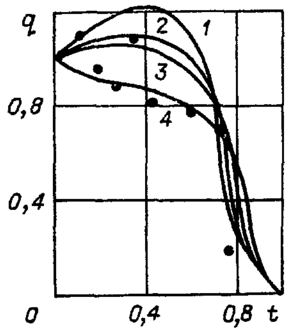

Fig. 5

4. Equation (3.6) applies if the number of cycles is sufficiently large. If this is not so, then on integrating (2.2) we have to allow for the motion of P_0 during the strain. According to (2.5) and (3.3), the equation for few-cycle fatigue with rigid strain can be written as

$$\delta = (p_*/N^{\alpha})(1-\beta). \tag{4.1}$$

Here p_* and α are defined by (3.8) and (3.9); the function β reflects the effects of the nonstationary damage accumulation and is as follows for $n = 1$:

$$\beta = 1 - \left[1 - (p_2/p_*)^2 \varkappa (L_*/p_2) - \frac{1}{p_*^2} S\right]^{1/2},$$

where $\varkappa(z) = 0.5 + z - z^2 \ln(1 + 1/z)$ and

$$S = \sum_{m=1}^{k^*-1} (-1)^{m-1} L_m L_{m-1} (1-\varkappa_m) \ln\left(1 + \frac{\delta}{L_m}\right) + \sum_{m=k^*}^{2N} L_m L_{m+1} [\varkappa_m + (1-\varkappa_m)\ln(1-\varkappa_m)],$$

$$L_m = L_* + p_2 + (m-1)\delta, \quad \varkappa_m = \frac{(-1)^m [L_*(p_1+p_2) + p_1 p_2]}{L_m L_{m+1}},$$

where k^* is the least odd number exceeding $(p_1^2 + 2L_* p_1)/\delta^2 - 1$. Then β (n, N, δ, L_*, p_2) is determined by numerical integration for $n \neq 1$. For $p_1/p_2 = \text{const}$ and $\delta \to 0$ we get $\beta \to 0$.

Equation (4.1) differs from the few-cycle fatigue equation derived for unsymmetrical rigid strain from (1.2) in giving a qualitatively correct reflection of the effects of the asymmetry on the working life. According to (4.1), the ratio of the working life in unsymmetrical strain ($R = p_1/p_2 = \text{const} \neq -1$) to that in symmetrical strain ($R = -1$) with the same scale δ tends to one as δ increases (increase in the working life). Also, within the framework of the model of (2.5) it is possible to regulate the effects of the asymmetry on the working life by means of L_*; the larger L_*, the greater the reduction in the working life because of the asymmetry under otherwise equal conditions. This is illustrated in Figs. 1 and 2, which give few-cycle fatigue curves derived from (4.1) for $L_*/D = 0.5$ (Fig. 1) and $L_*/D = 1$ (Fig. 2). Curve 1 corresponds to $R = -1$, curve 2 to $R = 0.5$, and curve 3 to $R = 0.75$. Figures 1 and 2 also show the experimental results of [20] for steel A302 and 5454-0 aluminum alloy correspondingly; the open points correspond to $R = -1$, the crosses to $R = 0.5$, and the filled points to $R = 0.75$. One concludes from Figs. 1 and 2 that $L_* \approx 0.5D$ for A302 steel, while $L_* \approx D$ for 5454-0 alloy.

5. Let the material be brought to the point of failure by monotone strain after a certain strain path terminating at point 0 in strain space (this corresponds to the undeformed state). At that point we consider the case where P_0 at the end of the preliminary strain path coincides with 0. Under these conditions, we can derive from (2.5) and (3.3) an equation relating the residual plasticity p^+ (the length of the segment of monotone strain to failure) and the damage Ω_0 due to the preliminary strain path:

$$\frac{\int\limits_0^{p^+/D}\left\{\frac{2\,[\gamma + L_0/D]\,\xi + \xi^2}{2\,(\gamma + L_0/D + \xi)}\right\}^n d\xi}{\int\limits_0^1\left[\frac{2\gamma\xi + \xi^2}{2\,(\gamma + \xi)}\right]^n d\xi} = 1 - \Omega_0. \tag{5.1}$$

According to (5.1), the residual plasticity is dependent not only on Ω_0, but also on the length of the preliminary strain path L_0, as it decreases as L_0 increases for $\Omega_0 = \text{const}$. This in part explains the large spread in the experimental data on the residual plasticity in coordinates $p^+ \sim (1 - \Omega_0)$ [20]. For $L_0 + L_* \gg D$, the dependence of p^+ on L_0 becomes unimportant, and (5.1) becomes

$$\frac{p^+}{D} = k\,(1 - \Omega_0)^{\alpha}, \quad k = \left\{(n+1)\int\limits_0^1\left[\frac{2\gamma\xi + \xi^2}{2\,(\gamma + \xi)}\right]^n d\xi\right\}^{\frac{1}{n+1}}. \tag{5.2}$$

An equation of the form of (5.2) was derived in [11] for the residual plasticity on the basis of (1.2) with $k = 1$ and $\alpha = 0.5$. Within the framework of (2.5), (5.2) gives a lower bound to p^+ for a fixed γ and a given Ω_0, while (5.1) gives an upper bound for $L_0 = 0$. The solid lines in Fig. 3 show curves corresponding to the upper and lower bounds to $p^+/D = \eta$ for $\alpha = \gamma = 0.5$. The points represent experimental results for A302 [20]. It is evident that the line given by (5.2) in fact sets a lower bound to the experimental data. On the other hand, many of the experimental points lie above the curve corresponding to (5.1) with $L_0 = 0$. Better agreement with the experimental data is obtained for lower values of α (see the dashed curves in Fig. 3, which correspond to $\alpha = 1/3$).

6. We now consider a case different that that in the previous section, namely where 0 does not coincide with P_0. The model of (2.5) then predicts the following effect: p^+ on deformation along the straight line OP_0 from 0 to P_0 should be larger than the residual plasticity p^- on deformation in the opposite sense. The following equations describe this effect for $n = 1$, which are derived for this case from (3.2) and (3.3):

$$p^+ = \rho_0 + \sqrt{\rho_0^2 + 2\mu\,(1 - \Omega_0) + r_1},$$

$$r_1 = \frac{1}{2}(p^+)^2 - 2p^+\rho_0 - (L_0 + L_*)\left(L_0 + L_* + p^+\right)[\varkappa + (1 - \varkappa)\ln(1 - \varkappa)], \quad p^- = -\rho_0 + \sqrt{\rho_0^2 + 2\mu\,(1 - \Omega_0) + r_2},$$

$$r_2 = \frac{1}{2}(p^-)^2 + 2\rho_0 p^- - (L_0 + L_*)\left\{p^- - [(L_0 + L_*) - 2\rho_0]\ln\left(1 + \frac{p^-}{L_0 + L_*}\right)\right\}. \tag{6.1}$$

Here $\mu = \frac{D^2}{4} + \frac{1}{2}\left[L_* D - L_*^2 \ln\left(1 + \frac{D}{L_*}\right)\right]$, $\varkappa = \frac{p^+ - 2\rho_0}{L_0 + L_* + p^+}$, is the distance between 0 and P_0. Figure 4 shows the dependence of the measure of this effect $\beta = (p^+ - p^-)/D$ on Ω_0 for the case

$L_* = 0$ and $\alpha = 0.5$ calculated from (6.1) for a preliminary strain path in the form of a single pulsating cycle. It also shows results obtained in the torsion of circular specimens of diameter 3 mm and length 60 mm in the working part made of LS-59 brass (points 1) or U-8 steel (points 2). Each experimental point has been obtained by averaging over five experiments with identical Ω_0. However, the condition $p^+ > p^-$ was obeyed even for each individual experiment, i.e., the effect was confirmed, at least for the materials tested. Figure 4 shows that the Ω_0 dependence of β is described in a qualitatively correct fashion by (2.5). We note that this effect does not occur within the scope of (1.2): The residual plasticity on deformation from the origin in strain space is not dependent on the disposition of the previous strain path.

In [22] we find experimental data indicating that the plasticity of steel in tension after minor strain in compression may be greater than the initial plasticity. This effect is described by (1.2). Figure 5 gives graphs for the dependence of the plasticity q_1 on preliminary plastic strain q_2 in the opposite sense derived from (2.5) and (3.3) ($q = q_1/D$, $t = q_2/D$). Curve 1 corresponds to $L_*/D = \infty$ and coincides with that obtained in [7], while curves 2-4 correspond to $L_*/D = 1$, 0.5, and 0.001, where we also show the experimental points from [22]. It is evident from the graph that the model (2.5) indicates that q_1 may exceed D for materials with sufficiently large L_*/D. Also, L_* may effect the magnitude of the effect, in accordance with the experimental data. For example, even the most upwardly deviant experimental points from [22] correspond better to curve 2 ($L_*/D = 1$) than to curve 1 ($L_*/D = \infty$).

Therefore, it is possible to extend the range of experimental data that can be described within the framework of a kinetic criterion for damage accumulation by incorporating the effects of the strain history on the damage accumulation rate in the form defined by (2.5).

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FORMULATION OF A DIAGNOSIS PROBLEM FOR A THERMOELASTIC MEDIUM

V. A. Lomazov and Yu. V. Nemirovskii

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By diagnosis problem we understand the determination of the characteristics of a medium from information obtained from a certain number of tests (test studies). Similar formulations are widely used in geophysics, particularly in seismic surveying. The general methods available have been discussed in [1]. Typical applications of these formulations and methods in diagnosis as regards the mechanics of deformable solids are related to the identification of unsatisfactory items, determining wear during use, and researching the effects of external factors on the properties of materials.

Here we deal with the determination of small changes in the thermoelastic characteristics of a material whose original properties are known. This can be interpreted as refining the properties of the material. In fact, when an item is manufactured, the material is subject to external factors arising from the production technology, which in general alter its properties. A method is proposed for determining the new thermoelastic characteristics on the assumption that these remain close to those of the medium that was originally homogeneous and isotropic. We consider an example of using this method.

1. The propagation of thermoelastic waves in an inhomogeneous anisotropic medium is described [2] by the following equations:

$$\rho\ddot{u}_i = (C_{ijkl}\, u_{k,l})_{,j} - (\beta_{ij}\Theta)_{,j}; \tag{1.1}$$

$$C_\varepsilon\dot{\Theta} - (K_{ij}\Theta_{,i})_{,j} = 0, \tag{1.2}$$

where ρ is density, Θ is relative temperature, $\mathbf{u} = (u_1, u_2, u_3)$ is the displacement vector, $\beta_{ij} = C_{ijkl}\alpha_{kl}$; α_{kl} are the thermal-expansion coefficient C_{ijkl} are the isothermal rigidity coefficients, and K_{ij} are the thermal conductivities. All of these quantities are functions of the spatial variables $\mathbf{x} = (x_1, x_2, x_3)$, $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$, $\Theta = \Theta(\mathbf{x}, t)$.

We denote by ρ^0, C^0_{ijkl}, β^0_{ij}, C^0_ε, K^0_{ij} the quantities characterizing the thermoelastic properties of a homogeneous isotropic medium. In that case, these quantities are constants, and the tensors C^0_{ijkl}, β^0_{ij}, K^0_{ij} have a specific (simpler) form [2].

In what follows we assume that the medium is weakly inhomogeneous and weakly anisotropic, i.e., the quantities $|\rho - \rho^0|$, $|C_\varepsilon - C^0_\varepsilon|$, $|C_{ijkl} - C^0_{ijkl}|$, $|\beta_{ij} - \beta^0_{ij}|$, $|K_{ij} - K^0_{ij}|$ have identical small orders

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